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SING PRIVILEGED INFORMATION TO MANIPULATE MARKETS:

INSIDERS, GURUS, AND CREDIBILITY

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by Roland BENABOU and Guy LAROQUE

ABSTRACT

We show how an insider, a financial journalist, or a "guru" can repeatedly manipulate an asset market to his own benefit, by making strategically distorted announencements or predictions. We also analyze the extent to which the public's attempt to learn over time whether or not this informed agent can be trusted may limit, in the long run, his influence on the market. In solving the model, we extend Sobel's [1985] results for repeated games of information transmission to the case where the sender has imperfect information, which generates richer and more realistic reputation processes over an infinite horizon.

1. Introduction

> "In the nervous market of 1987, Mr. P... has emerged as both prophet and deity, an adviser whose advice reaches so many investors that he tends to pull the market the way he has predicted it will move.... Mr. P...'s words carry such power because he appears to have called the broad outlines of the bull market right since 1982, although he has made some bad calls on short-term moves. Analysts who rank market newsletters have consistently rated his forecasts as the most accurate. But as recently as September, he was still telling his subscribers to buy stocks."
> (International Herald Tribune, October 3, [1987])

This and similar well-known cases of financial analysts, chief economists or specialized columnists whose predictions about prices, interest rates, etc., generate such faith that they influence the market raise a double question.

First, what privileged information or techniques provide--or are believed by so many people to provide--these individuals with superior knowledge of future market evolutions? Secondly, given that they have, or are thought to have such knowledge, why should they be expected to report it honestly rather than manipulate the information which they feed the market to their own advantage?

The first question has received substantial theoretical and empirical attention, since the efficient markets hypothesis stipulates that no profitable knowledge can be gained by analyzing publicly available information, whether on past prices or on fundamentals (weak or semi-strong forms), or even by making use of privileged "inside" information (strong form). This paper addresses the second question, namely the credibility of announcements, articles etc., issued by

individuals who are known or believed to have privileged information on future returns, and may have incentives to use this fact to manipulate the market. It is thus related both to the literature on insider trading and to that on games of strategic communication. This double relationship is discussed below.

Many recent papers have emphasized the revelation of private information through prices. They examined the incentives which informed market participants have to restrain their trades, so as to limit revelation through prices, and the returns to private information.

A competitive market (Grossman-Stiglitz [1980]) provides no incentives for self-restraint. As a result, an informational equilibrium may fail to exist, unless there are enough noise traders to prevent prices from being too revealing. If individual traders retain some monopoly power, on the contrary, they will restrain their informed trading (Kyle [1986]), or spread it over time (Kyle [1985]) so as to limit, or delay the leakage of their information through the price. The equilibrium will be less (or more slowly) revealing than in the competitive case, and generate positive profits from informed trading (see also Laffont-Maskin [1988]). In these models, the privately informed agents can trade on the market, but are not allowed to disseminate information in any other way. Admati and Pfleiderer [1986] examine the dual case of an informational monopolist who is not allowed to trade himself, but can sell information to market participants. He will then internalize the externality inherent to informed trading, and limit its extent by: a) controling, through the price he charges, how many agents become informed; b) lowering the accuracy of the information he provides, by adding to it an unbiased random noise.

In this paper we take the idea of altering the information

communicated to the market one step further, by considering the case where the privately informed agent can both speculate and (costlessly) disseminate information (or disinformation), in the form of articles, newsletters, forecasts, "tips", etc. The emphasis is shifted away from the revelation of information through prices (the informed agent's trades are assumed to have a negligible effect on the market)[1] and on to the substantially different questions of whether he should be believed, and of the extent to which he can influence the market, as in the initial quotation.

Indeed, the informed agent now cares about the market price per se. As a result, his incentive is no longer to simply decrease the average accuracy of his message by adding an unbiased noise to his signal, as in Admati-Pfleiderer [1986], but to mislead the market by communicating a biased message. For instance, if this agent learns that the return on some asset is likely to be high, he can, as noted by Hirschleifer [1971] buy large quantities of it, then publicize his information; but to the extent that such announcements are indeed believed, he can do even better by forecasting a low return, then buying the valuable asset at a depressed price.[2]

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[1] We are thus interested in the consequences of secret trading which remains undetected. More generally, one could assume that the insider is at least partially able to "hide" behind noise traders, as in Kyle [1985], [1986], or to induce a pooling equilibrium, as in Laffont-Maskin [1988].

[2] Jaffe [1974] relates the following case, which led to prosecution by the Securities and Exchange Commision. In late 1963, drillings by the Texas Gulf Sulphur Company struck huge mineral deposits. Between November 1963 and mid-April 1964, company officials engaged in a large-scale effort to convince the public of the opposite by falsifying evidence (such as drill cores), while accumulating company shares and options. On April 12, 1964, they even issued a press release stating that the technical evidence was inconclusive; four days (and many thousand shares) later, the company admitted that deposits had in fact been found.

The potential for such manipulation of information is reflected in U.S. securities laws:

> The key statutory provision is rule 10(6)(5) of the securities law. As originally drafted in 1942, this was thought to apply against officers of a company who defrauded shareholders...for example by buying shares for themselves...<u>after</u> making intentionally misleading earnings forecasts.
>
> (<u>The Economist</u>, [1987]; emphasis added).

The problem we focus on in this paper is therefore one of strategic information transmission, as in Crawford and Sobel [1982] or Sobel [1985], in which a "sender" observes the state of nature, then (costlessly) transmits a message to a receiver who responds by taking an action which, together with the state of nature, determines the welfare of both.

The differences with our model fall into two categories. First, the game which determines the payoffs is here a simple model of an asset market, so that there is continuum of receivers, (the public), and it is the sum of their actions (demands), materialized in the market-clearing price, which matters for the sender; moreover, this aggregate reaction may be random due to noise traders.

The second difference is more substantive, both technically and for the realism of the results: the sender's signal only provides him with <u>imperfect information</u> on the true state of the world (value of the asset) so that a wrong prediction cannot be unambiguously attributed to dishonesty. Thus, in a context of repeated interaction (Kreps-Wilson [1982], Milgrom-Roberts [1982], and especially Sobel [1985]) his reputation and credibility fluctuate over time, according to a process which he controls only partially. Moreover, this process is smoother

and more realistic than those encountered in the above-mentioned papers, where reputation typically increases until the privately informed agent makes a (fully revealing) opportunistic move, after which it is lost forever. In that respect our model presents a similarity with Holmström's [1982] model of repeated moral hazard with learning about ability, and with Cuckierman and Meltzer's [1986] model of reputation and ambiguity about a government's changing preferences.

Section 2 presents a simple model of an asset market with announcements by an informed agent, and relates the latter's payoff to his credibility. Section 3 examines the inference process through which uninformed agents assess his credibility, and the resulting equilibrium outcome. In Section 4, the game is repeated indefinitely; uninformed agents attempt to learn over time whether or not the sender is honest, while the latter takes account of this learning in his decision problem. A unique (symmetric) sequential equilibrium is shown to exist, then analyzed. In Section 5, the stochastic properties of the equilibrium reputation process are established. Finally, Section 6 presents various extensions of the basic model. All proofs are gathered in Section 7.

While presented here in the context of an asset market, the results concerning the existence, uniqueness, and stochastic properties of equilibrium hold for all games of repeated information transmission with payoffs satisfying appropriate conditions. They are therefore potentially applicable to other problems, such as information-sharing by oligopolists who do not commit to noisy reporting rules (as they do in Gal-Or [1985] or Vives [1985]) but choose each report optimally.

2. The short-run model

We present here a highly simplified model of a financial market where an informed agent may both secretly trade on the basis of his private knowledge, and disseminate true or false information at no cost.[3] This agent could be a corporate executive making announcements about his company; a stockbroker issuing portfolio recommendations; an influential "guru" making predictions or publishing a newsletter; or a journalist writing a financial column.

2a. The framework

There is a continuum of agents, indexed by a in $A = [0,1]$, endowed with the Lebesgue measure. During the period of reference, a financial asset is traded. The return on this asset is contingent upon the realization of a random variable $\tilde{n}$, which becomes publicly observable at the end of the period ($\tau = 1$). At the beginning of the period ($\tau = 0$), a single agent, say $a = 0$--hereafter often referred to as "the journalist"--privately observes a random signal $\tilde{s}$, correlated with $\tilde{n}$. At some time ($\tau = 1/2$) during the trading period, he can send to some of the other participants, a in $(0,1]$--hereafter referred to as "the public"--a message $\tilde{m}$ chosen from some message space $\underline{M}$.

To simplify, we assume that both the state of nature $\tilde{n}$ and the insider's private signal $\tilde{s}$ take values in $N = \{-1,+1\}$. The asset pays \$1 if $\tilde{n} = +1$, zero otherwise. The stochastic properties of the

[3] Ours is therefore a model of costless signalling, or "cheap talk". Allowing for costs of transmitting messages, or for the insider to set a price for his information, would increase his opportunities for signalling his type (honest or opportunistic) but need not result in a completely separating equilibrium. The assumptions of the paper are also quite realistic for a financial journalist, who controls neither the readership nor the price of the newspaper he writes his column in.

couple $(\tilde{s},\tilde{n})$ are common knowledge at the outset of the period (τ = 0) and described by:[4]

$$(2.1)\quad \begin{cases} \text{Prob}[\tilde{s} = -1] = \text{Prob}[\tilde{s} = +1] = 1/2 \\ \text{Prob}[\tilde{n} = -1] = \text{Prob}[\tilde{n} = +1] = 1/2 \\ \text{Prob}[\tilde{n} = n|\tilde{s} = n] = p \quad \text{for all } n \text{ in } N \\ \text{Prob}[\tilde{n} = n|\tilde{s} = -n] = 1\text{-}p \text{ for all } n \text{ in } N, \end{cases}$$

where $1/2 < p < 1$. Let $p^+ = p$, $p^- = 1-p$. By (2.1), $\text{Prob}[\tilde{n}=n|\tilde{s}=s]$ only depends on the product $sn \in \{-1,1\}$. We will therefore denote this probability as p^{sn}, identifying for brevity +1 with "+" and -1 with "-".

The timing of signals and moves is summarized in Figure 1. All agents maximize the expected utility of their wealth at the end of the period, given their information.

Assumption A1: For all a in A, agent a's utility function u_a: $R_+ \to R$ is twice continuously differentiable, increasing, concave, with $\lim_{x \to +\infty} u_a'(x) = 0$ and $\lim_{x \to 0} u_a'(x) = +\infty$. Furthermore, u_a exhibits non-increasing absolute risk-aversion. Finally, both marginal utility u_a' and initial wealth $w_a > 0$ are continuous in a on A.

To ensure that agents' demand curves for the risky asset slope downward, and to simplify the model in general, we shall concentrate on the following case:

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[4] Apart from those of Section 5, the paper's results actually apply to a "pure guru" as well as to a true insider, i.e. require only that all agents, including agent zero, believe with probability one that $\tilde{s}$ is correlated with $\tilde{n}$ as in (2.1), even if in reality $\text{Prob}[\tilde{n} = n|\tilde{s} = n] = 1/2$.

Assumption A2 (purely speculative market): The total supply of the asset, as well as each agent's initial holding of it, are zero.

2b. The market after broadcast of the message

Let Ω_a denote agent a's available information at $\tau = 0$. At $\tau = 1/2$, after receiving the journalist's message, his beliefs over the future value of the asset are described by:

(2.2) $\quad \pi_a = \text{Prob}[\tilde{n} = 1 | \tilde{m} = m, \Omega_a]$.

Given (2.1), we have for the journalist: $\pi_0 = p^s$. We assume that the agents in the public fall into two categories. First there are rational, well-informed speculators, a in $(0,\alpha]$, $0 < \alpha \leq 1$, who use the message $\tilde{m}$ sent by the journalist, their prior probability on its truthfulness, and Bayes' rule, to revise their priors on $\tilde{n} = 1$ from one half to a common belief $\pi_a = \pi^{\tilde{m}}$. The specifics of this updating, which crucially involves the journalist's credibility, are examined in the next section; in particular, $\pi^{\tilde{m}}$ is shown to lie in $[1-p, p]$. Then there are noise traders, or, more properly speaking, speculators with noisy information, a in $(\alpha,1]$, ($\alpha = 1$ is not ruled out), who do not correctly receive or take into account the message $\tilde{m}$, and act according to a common belief $\pi_a = \tilde{\pi}$, where $\tilde{\pi}$ is drawn at $\tau = 1/2$ from a given, common knowledge distribution with support in $[1-p,p]$.[5] Thus $\tau = 1/2$ is the time at which public information, whether provided by the journalist or extraneous, arrives in the market and is

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[5] This is simply a convenient way, in this model, of introducing a random component in the supply of the asset to the market. In general there could be a shock in both subperiods, i.e. at $\tau = 0$ as well as $\tau = 1/2$; we focus here on the innovation in noise, or supply, which interferes with the journalists' message.

incorporated into beliefs and prices (see Figure 1).

For the moment, we shall treat rational speculators' common belief $\pi_a = \tilde{\pi}^m$ as a parameter, and simply assume:

Assumption A3: For all a in A, π_a belongs to $[1-p,p]$.

The market clearing price for the asset at $\tau = 1/2$ will depend on the beliefs π_a, a in A. From the point of view of the journalist at $\tau = 0$, when he knows which value $\pi = \pi^m$ he plans to induce, but not which value of $\tilde{\pi}$ will occur, this price is a random variable, which we denote by $\tilde{P}(\pi)$, with cumulative distribution M_π. Let P be the set of random variables with support in $[1-p,p]$. We have:

Property 2.1: Under assumptions A1 to A3, for every π in $[1-p,p]$ there exists a unique market clearing price $\tilde{P}(\pi)$ in P. Moreover, if $\pi > \pi'$ then $M_\pi \leq M_{\pi'}$.

The market-clearing price thus increases (stochastically) with the belief $\tilde{\pi}$ about the likelihood of the "good" state which the informed agent is able to induce among rational speculators.

2c. The two types of market manipulation

As shown by Figure 2, the market offers in each of the two subperiods a different speculative opportunity for a privately informed agent who has some credibility, i.e. some influence on the public's beliefs. The first one, which we shall call "pre-announcement speculation", consist of trading during the first subperiod in anticipation of the effect of the planned announcement at $\tau = 1/2$; for

instance, the journalist can buy the asset, then announce some good news ($\tilde{m} = 1$) on its future value, so as to raise its price and the value of his portfolio at $\tau = 1/2$. Note that the profitability of this scheme depends solely on the announcement's credibility, and not on the actual information which is in the hands of the journalist. The second scheme, called "post-announcement speculation", consists of inducing erroneous beliefs by a misleading announcement at $\tau = 1/2$, then trading on the basis of the private information, the value of which is increased by the fact that the asset is now even more incorrectly priced. Thus, if $\tilde{s} = 1$, indicating a valuable asset, the journalist can depress its price at $\tau = 1/2$ by announcing $\tilde{m} = -1$, then buy it at very advantageous terms and expect to realize large profits at $\tau = 1$.

The two schemes can also be combined for greater profits, as follows: if $\tilde{s} = -1$ (say), buy at $\tau = 0$, announce $\tilde{m} = +1$ at $\tau = 1/2$, then sell short.

The first type of scheme was considered by Hirschleifer [1971], while the rule from the U.S. securities law quoted earlier originally focused on the second one. Actual occurrences, on a large scale, of these two types of market manipulation are best illustrated and contrasted by the following examples:[6]

Pre-announcement speculation: in 1814 a group of people secretly bought large numbers of British government securities, then arranged for false reports of the death of Napoleon to reach London; the price rose sharply, and they made large profits by selling out their holdings (Rex vs. de Berenger, quoted by King [1977]).

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6 We are indebted to Mervyn King for providing us with these examples.

Post-announcement speculation: during the battle of Waterloo (1815), the banker Nathan Rothschild, who was known to have superior information from the continent due to a system of messenger pigeons, walked around the City looking dejected, spreading the news that the battle was going badly, and had agents of his openly sell British government securities. Meanwhile, he was secretly buying much larger quantities of these securities, taking advantage of the depressed price and of his actual knowledge of an impending victory.

As mentioned above, the second scheme is more closely linked to informational considerations than the first one. Note also from Figure 2 that the expected return is larger under the second scheme than under the first: $R_{2L} > \max\ [R_{1T}, R_{1L}]$ (riskiness, however, may go in the same or in the opposite direction). For these reasons, but essentially because it makes the analysis much more tractable, we will mostly focus on the second scheme, by assuming:

Assumption A4: The journalist cannot trade before $\tau = 1/2$.

This restriction will be maintained until Section 6b, where we shall examine under which conditions allowing pre-announcement as well as post-announcement speculation leaves the results unchanged.

2d. The journalist's expected gains from trade.

Let us now compute the expected utility of the journalist, accruing from his trades on the market, as a function of the signal s he received and of the message m he plans to send at $\tau = 1/2$. Since it only depends on m through the beliefs $\pi = \pi^m$ induced in the public, it

will be denoted as $U(s,\pi)$:

$$(2.3)\quad U(s,\pi) = \int_{1-p}^{p} \max_{B \in R} \{p^s u_0(w_0 + (1-P)B) + (1-p^s)u_0(w_0 - PB)\}\, dM_\pi(P)$$

The journalist's incentive to manipulate the price is clearcut. For instance if $\tilde{s} = 1$, he is interested in inducing the lowest possible price, so as to buy the asset for cheap and make large profits at $\tau = 1$. By Property 2.1, this means that he will send a message that induces a low value of π.

Property 2.2: Under assumptions A1 to A4, for every s in N, $s.U(s,.)$ is a continuous and strictly decreasing function from $[1-p,p]$ into R.

The interests of the agent who sends the message are thus diametrically opposed to those of the receivers, as in Sobel [1985], and in contrast to Crawford and Sobel [1982], where the disagreement concerns only the degree with which both parties want to move in a common direction. This property is a central ingredient in the paper's results. When studying the repeated game in Section 4, it will be useful to combine it with a symmetry assumption on the noisy beliefs, or information, of the traders a in $(\alpha,1]$ (if any).

Assumption A5 ("symmetric noise"): The price distributions M_π and $M_{1-\pi}$ are symmetric with respect to one another around one-half:

$$M_\pi(P) + M_{1-\pi}(1-P) = 1$$

for all π in $[1-p,p]$ and P in $[0,1]$.

When there are no noise traders, this assumption always holds ($\bar{P}(\pi) \equiv \pi$). Alternatively, with common logarithmic utility functions,

it can be shown that: $\hat{P}(\pi) = \alpha\, \pi + (1-\alpha)\, \bar{\pi}$, while with common CARA utility functions: $\hat{P}(\pi) = [1 + ((1-\pi)/\pi)^{\alpha}((1-\bar{\pi})/\bar{\pi}))^{1-\alpha}]^{-1}$. In both cases, if $\bar{\pi}$ is distributed uniformly on $[1-p,p]$, $M_{\pi}(P)$ satisfies assumption A5, as well as Property 2.1.

Property 2.3: Under assumptions A1 to A5, the journalist's expected utility $U(s,\pi)$ satisfies the following symmetry property:

$$U(-s,\pi) = U(s,1-\pi), \text{ for all } s \text{ in } N \text{ and } \pi \text{ in } [1-p,p].$$

3. Information transmission and the single period game.

It should be clear from the preceding section that our model can be considered as a game of strategic information transmission, as in Crawford-Sobel [1982] and particularly Sobel [1985], in which we allow the sender's information $\bar{s}$ to be noisy. All our results remain valid for any such game in which the function $U(s,\pi)$, denoting the sender's expected utility when he has observed the signal s and sent a message which induces the belief π, satisfies Property 2.2 and, when specified, Property 2.3.

An important feature of many of these models is that the receiver is uncertain about the sender's "honesty". More precisely here, we will assume that the public has prior probability ρ that the journalist always truthfully announces $\bar{m} = \bar{s}$, and $1-\rho$ that he opportunistically maximizes his expected utility gains from trade, manipulating information to his advantage. As will be seen below, Property 2.2 precludes information from being transmitted in equilibrium unless the public's prior ρ is high enough (in the one-shot game) or at least positive (in the infinitely repeated game). "Honesty" can be given the following "behavioral" interpretation: an honest journalist is

one who highly values and therefore strictly adheres to a code of ethics under which he pledges to always tell the truth. Another interpretation, in terms of payoff uncertainty, is that a journalist who is caught lying faces penalties, which are to be substracted from $U(s,\pi)$, but that the public is uncertain whether the expected penalty, taking account of the privateky known probability of escaping discovery, is high enough or not to deter lying. Alternatively, the unknown characteristic of the journalist could be whether or not he is able to carry out trades without being fully detected. In such cases, "honesty" is the consequence of rational behavior.

There are (at least) two equivalent ways of describing the transmission of information from the journalist to the public. First, the game-theoretic approach. The message space is $\underline{M} = N$ itself, and the sender plays a mixed strategy described by a signalling rule $q^{(\cdot)}: N \rightarrow [0,1]$, which associates to each observed signal s the probability with which it is truthfully announced:

$$(3.1) \qquad q^s = \text{Prob}[\tilde{m} = s | \tilde{s} = s].$$

The honest sender always plays $q^s = 1$, and we shall reserve the notation q^s for the opportunistic type. As above for p^s, we shall abbreviate q^{+1} and q^{-1} as q^+ and q^-.

Rational speculators do not know the value of $\tilde{s}$, but in equilibrium they are aware of the signalling rule used by the opportunistic journalist. Their (common) available information $\Omega_a = \Omega$ therefore includes $q^{(\cdot)}$, as well as their beliefs $(\rho, 1-\rho)$ over the journalist's being honest (H) or opportunistic (O). Given a message $\tilde{m} = m$, they update their priors on $\tilde{n}$, using Bayes' rule:

$$(3.2) \qquad \mathrm{Prob}[\tilde{n}=n|\tilde{m}=m,\ \Omega] = \frac{\mathrm{Prob}[\tilde{n}=n,\ \tilde{m}=m|\Omega]}{\mathrm{Prob}[\tilde{m}=m|\Omega]}$$

$$= \frac{\rho\ \mathrm{Prob}[\tilde{n}=n,\ \tilde{m}=m|\Omega,H] + (1-\rho)\ \mathrm{Prob}[\tilde{n}=n,\tilde{m}=m|\Omega,0]}{\rho/2 + (1-\rho)[q^m + 1-q^{-m}]/2}$$

$$= \frac{\rho\ p^{mn} + (1-\rho)[q^m\ p^{mn} + (1-q^{-m})\ p^{-mn}]}{\rho + (1-\rho)\ [q^m + 1-q^{-m}]}$$

These updated beliefs are summarized by the couple $(\pi^+,\ \pi^-)$, where:

$$(3.3) \qquad \pi^m = \mathrm{Prob}[\tilde{n} = 1|\tilde{m} = m], \qquad m \text{ in } N.$$

Given the signal he observed and the inference process (3.2) of rational speculators, the opportunistic journalist chooses his message as a Stackelberg leader vis-a-vis the public, while the latter is conscious of this behavior when assessing the message's credibility and updating the outcome probabilities. Hence:

<u>Definition 3.1</u>: <u>An equilibrium of the single period game is a couple</u> $(q^+,\ q^-)$ in $[0,1] \times [0,1]$ <u>such that</u>:

(i) <u>for all</u> s <u>in</u> N, q^s <u>maximizes</u> $q'\ U(s,\pi^s) + (1-q')\ U(s,\pi^{-s})$

<u>over</u> q' <u>in</u> $[0,1]$, <u>where, for all</u> m <u>in</u> N:

$$(ii) \quad \pi^m = \frac{\rho\ p^m + (1-\rho)\ [q^m\ p^m + (1-q^{-m})(1-p^m)]}{\rho + (1-\rho)[q^m + 1-q^{-m}]}\ .$$

Note that π^m always lies in $[1-p,p]$, in conformity with A3. It is also possible to subsume the signalling rule $q^{(\cdot)}$ into beliefs, and give a definition similar to that of a rational expectations equilibrium. Given a message m, all rational agents (including the journalist) expect a price distribution $\tilde{P}(\pi^m)$. For this expectation to be correct, the probability which the informed public infers (and then uses in its portfolio decision) from $\tilde{m} = m$ and its knowledge of

the opportunistic journalist's maximizing behavior must be π^m. This fixed-point requirement can be expressed as follows.

Definition 3.2: An equilibrium of the single period game is a couple (π^+, π^-) in $[1-p, p]$, such that for all m in N:

$$\pi^m = \rho\, p^m + (1-\rho)\ \mathrm{Prob}[\tilde{n} = 1 | \pi^m \in \mathrm{argmax}(U(\tilde{s}, \pi^n),\ n \in N)]$$

The equivalence between the two definitions is immediate (the latter amounts to using $[1-p, p]$ instead of $N = \{-1, +1\}$ as the message space). Property 2.2 implies the following result (as in Sobel [1985]):

Proposition 3.1: Under assumptions A1 to A4, the single period game has a unique equilibrium.
If $\rho \le 1/2$, there is no transmission of information: $\pi^+ = \pi^- = 1/2$.
If $\rho > 1/2$, the message is believed to convey information although the opportunistic journalist always lies:

$$\pi^+ = \rho\, p + (1-\rho)\ (1-p) > \rho\ (1-p) + (1-\rho)\ p = \pi^-.$$

4. The infinite horizon reputation game

In the single period game, the reputation ρ of the journalist was given a priori. One of our main purposes is to examine how this reputation evolves over time when the game is repeated.

The intertemporal setup is the following. At the beginning of the game, $t = 1$, a journalist is selected randomly. It is known that with probability ρ_1 he is honest and that with probability $1-\rho_1$ he is opportunistic. The journalist chosen at date 1 stays in charge of the journal in all the future periods, $t = 1, 2, \ldots$[7] When opportunistic, he maximizes the discounted (at rate β) expected sum of instantaneous

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7 The stochastic renewal of the journalist is considered in Section 5b.

utilities. On the other hand, at each date, a new "generation" of speculators enter the market (with constant utilities and wealth over time), trade the financial asset in the same context as in the one-period game, and finally exit to consume its yield. The only intertemporal link is the journalist's reputation, which is revised at the end of every period, depending on whether his forecast is realized or not, and left with the next generation (more formally, we study the sequential equilibria of the intertemporal game in Markov strategies).[8]

4a. Updating the reputation

As in Section 2b, let Ω_t be the information of the public at the beginning of period t. By definition, the journalist' reputation is:

$$(4.1) \qquad \rho_t = \mathrm{Prob}[H|\Omega_t].$$

During period t, a game similar to that of Section 3 is played. A signal $\tilde{s}_t = s$ is received by the journalist, who then sends a message $\tilde{m}_t = m$, using a signalling rule $q_t^{(\cdot)}$. Rational speculators then compute, according to (3.2), the probability:

$$\pi_t^m = \mathrm{Prob}[\tilde{n}_t = 1|\tilde{m}_t = m, \Omega_t],$$

where, as before, the equilibrium signalling rule $q_t^{(\cdot)}$ of the

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8 Our game thus belongs to the class of games with one long-run and a sequence of short-run players considered by Fudenberg-Kreps-Maskin [1987] and Fudenberg-Levine [1988]. These authors provide very general bounds for the limits of equilibrium payoffs when the discount factor β goes to one, by focusing on certain "committment-type" strategies. We consider a more restrictive class of games, but provide results on the equilibrium itself, for any β. Note finally that both $\bar{m} = 1$ and $\bar{m} = -1$ always have positive probability (when $\rho > 0$) so that a sequential equilibrium is here simply a Bayesian perfect equilibrium.

opportunistic journalist is part of their information Ω_t. Finally, the outcome of $\tilde{n}_t$ is observed, and at the beginning of period t + 1, the journalist's reputation is updated:

$$(4.2) \qquad \rho_{t+1} = \text{Prob}[H|\Omega_{t+1}] = \text{Prob}[H|\tilde{m}_t = m,\ \tilde{n}_t = n,\ \Omega_t]$$

By Bayes' formula:

$$\rho_{t+1} = \frac{\text{Prob}[H,\ \tilde{m}_t = m,\ \tilde{n}_t = n|\Omega_t]}{\text{Prob}[\tilde{m}_t = m,\ \tilde{n}_t = n|\Omega_t]}$$

or:

$$(4.3) \qquad \rho_{t+1} = \frac{\text{Prob}[\tilde{n}_t = n|\tilde{m}_t = m,\ H].\text{Prob}[\tilde{m}_t = m|H].\text{Prob}[H|\Omega_t]}{\text{Prob}[\tilde{m}_t = m,\ \tilde{n}_t = n|\Omega_t]}$$

The numerator of (4.3) is equal to $\rho_t.p^{mn}/2$, while the denominator is equal to the numerator in (3.2). Hence:

$$(4.4) \qquad \rho_{t+1} = \frac{p^{mn}}{\rho_t\ p^{mn} + (1-\rho_t)[q_t^m\ p^{mn} + (1-q_t^{-m})(1-p^{mn})]}\ \rho_t$$

If the sender has perfect information ($p^+ = p = 1$), as in Sobel [1985], his reputation improves monotonically until the first time he sends an incorrect message, after which $\rho_t = 0$ forever (because $p^- = 0$). When the signal he receives is itself noisy ($p < 1$), the evolution of his reputation is smoother, because his prediction may turn out to be wrong with positive probability even when he reports truthfully. Thus $\rho_t > 0$ implies $\rho_{t+1} > 0$ even if $m_t \neq n_t$.

Since ρ_t embodies all the relevant information available to the public at the beginning of period t, it follows, from their (subjective) point of view, a martingale:

$$(4.5) \qquad E[\rho_{t+1}|\Omega_t] = \sum_{m,n} \text{Prob}[H|\tilde{m}_t=m,\ \tilde{n}_t=n,\ \Omega_t] \times \text{Prob}[\tilde{m}_t=m,\ \tilde{n}_t=n|\Omega_t] = \text{Prob}[H|\Omega_t] = \rho_t.$$

The actual (objective) stochastic process of ρ_t along an equilibrium path will depend on which type of journalist is actually playing; it will be characterized in Section 5.

4b. Symmetric equilibrium in the reputation game.

When choosing his messages, the opportunistic journalist must now take into account their effect on his future reputation, as well as on his short-run profit. Since future reputations depend on the current one (by (4.4)), his (stationary) strategy will be a function q: $[0,1] \times N \to [0,1]$ of this state variable, and subsequently so will the beliefs π: $[0,1] \times N \to [1-p,p]$ of rational speculators: if the reputation at date t is ρ_t, a signal $\tilde{s}_t = s$ is reported truthfully with probability $q_t^s = q^s(\rho_t)$, and the credibility of a message $\tilde{m}_t = m$ is $\pi^m(\rho_t)$.

In order to keep the problem tractable, we will concentrate, from now on, on the symmetric equilibria of the symmetric game. We assume that Property 2.3 is satisfied (for instance because the symmetric noise assumption A4 holds), i.e.:

$$U(-s,\pi) = U(s,1-\pi), \text{ for all } s \text{ in } N \text{ and } \pi \text{ in } [1-p,p],$$

and we only consider strategies such that the probability with which $\tilde{s}$ is truthfully reported is independent of its particular realization:

$$(4.6) \qquad q^+(\rho) = q^-(\rho), \qquad \text{for all } \rho \text{ in } [0,1].$$

We shall denote it by $q(\rho)$. In any given period (for which we omit the time subscript), the probabilities of the final outcome inferred by the public, given by (3.1), simplify to:

$$(4.7) \qquad \text{Prob}[\tilde{n}=n|\tilde{m}=m,\Omega] = \rho\, p^{mn} + (1-\rho)[q(\rho)p^{mn} + (1-q(\rho))p^{-mn})].$$

This expression only depends on the product mn, so that the probability that $\tilde{n} = +1$ when $\tilde{m} = +1$ is equal to the probability that $\tilde{n} = -1$ when $\tilde{m} = -1$, i.e.:

$$(4.8) \qquad \pi^{+}(\rho) = \rho\, p + (1-\rho)\, [p\, q(\rho) + (1-p)\, (1-q(\rho))] = 1 - \pi^{-}(\rho)$$

The public's beliefs are thus fully described by $\pi(\rho) \equiv \pi^{+}(\rho)$, which represents the reliability, or credibility, of any message from the journalist, i.e. the probability with which it is expected to be realized. Finally, the reputation updating formula (4.4) becomes:

$$(4.9) \qquad \rho_{t+1} = \begin{cases} \rho^{+}_{t+1} = \dfrac{p\rho_t}{\pi_t} & \text{if } m_t = n_t \\ \rho^{-}_{t+1} = \dfrac{(1-p)\rho_t}{1-\pi_t} & \text{if } m_t \neq n_t \end{cases}$$

where $\pi_t \equiv \pi(\rho_t)$. Since π_t is in $[1-p,p]$, the journalist's reputation strictly increases (resp. decreases) each time his prediction turns out to be correct (resp. incorrect), unless he is known to be honest or dishonest for sure ($\rho_t = 0$ or $\rho_t = 1$), or to act honestly ($q_t = 1$).[9]

We are now in a position to transpose Definition 3.1 to the symmetric reputation game. Consider the opportunistic journalist, with a reputation ρ, when he has received the signal $\tilde{s} = 1$ and chooses a

[9] One source of difficulty in the non-symmetric case is that the inequality $\rho^{-}_{t+1} < \rho_t < \rho^{+}_{t+1}$ need not hold.

message $\tilde{m}$. If he says the truth, $\tilde{m} = 1$, the public will believe $\tilde{n} = 1$ to occur with probability $\pi(\rho)$. Also, with probability p, his prediction will be correct, so that by (4.9) his reputation at date t + 1 will increase to $p\rho/\pi(\rho)$; with probability 1-p, it will be wrong and his reputation will decrease to $(1-p)\rho/(1-\pi(\rho))$. Thus if $W(\rho)$ is the discounted expected sum of the journalist's utility at the beginning of a period where his reputation is ρ, he gets, when he chooses $\tilde{m} = 1 = \tilde{s}$ (the index T stands for truth):

$$(4.10) \qquad V_T(\rho) = U(1,\pi(\rho)) + \beta p\, W\left[\frac{p\rho}{\pi(\rho)}\right] + \beta(1-p)\, W\left[\frac{(1-p)\rho}{1-\pi(\rho)}\right].$$

Similarly, if he announces $\tilde{m} = -1 = -\tilde{s}$ (L stands for lie):

$$(4.11) \qquad V_L(\rho) = U(1,1-\pi(\rho)) + \beta p\, W\left[\frac{(1-p)\rho}{1-\pi(\rho)}\right] + \beta(1-p)\, W\left[\frac{p\rho}{\pi(\rho)}\right].$$

By symmetry, when $\tilde{s} = -1$, he also gets $V_T(\rho)$ if he says the truth and $V_L(\rho)$ if he lies. In both states, the trade-off between his short-run incentive to lie and his concern for reputation is therefore expressed by the difference:

$$(4.12) \qquad \begin{aligned} V_T(\rho) - V_L(\rho) &= U(1,\pi(\rho)) - U(1,1-\pi(\rho)) \\ &\quad + \beta\,(2p-1)\left[W\left[\frac{p\rho}{\pi(\rho)}\right] - W\left[\frac{(1-p)\rho}{1-\pi(\rho)}\right] \right] \end{aligned}$$

and he chooses his message at the beginning of the period so as to achieve the intertemporal utility max $[V_T(\rho), V_L(\rho)]$. Hence:

Definition 4.2: A symmetric equilibrium of the repeated game is a function $q:[0,1] \to [0,1]$, for which here exists $W: [0,1] \to R$ such that:

(i) For all ρ, $q(\rho)$ maximizes $q' V_T(\rho) + (1-q') V_L(\rho)$

over q' in $[0,1]$, with $V_T(\rho)$ and $V_L(\rho)$ given by (4.10)-(4.11), and:

(ii) $\quad \pi(\rho) = \rho p + (1-\rho)[pq(\rho) + (1-p)(1-q(\rho))]$.

(iii) $\quad W(\rho) = \text{Max}\ [V_T(\rho), V_L(\rho)]$.

Here again, it is convenient to eliminate $q(\rho)$ and characterize the equilibrium in terms of the public's belief $\pi(\rho)$ only, as in Definition 3.2. By (i) and (ii) above, when $V_T(\rho) > V_L(\rho)$, $q(\rho) = 1$ so $\pi(\rho) = p$. When $V_T(\rho) < V_L(\rho)$, $q(\rho) = 0$ so $\pi(\rho) = \rho p + (1-\rho)(1-p)$. Finally, when $V_T(\rho) = V_L(\rho)$, $\pi(\rho)$ takes value in $[\rho p+(1-\rho)(1-p), p]$. Thus:

Proposition 4.3: A symmetric equilibrium of the repeated game is characterized by a function $\pi: [0,1] \to [1-p, p]$ such that there exists $W: [0,1] \to R$ with:

(i) Defining $V_T(\rho)$ and $V_L(\rho)$ as in (4.10) and (4.11):

$$
(4.13) \begin{cases} V_T(\rho) > V_L(\rho) & \text{implies } \pi(\rho) = \rho, \\ V_T(\rho) = V_L(\rho) & \text{implies } \rho p + (1-\rho)(1-p) \leq \pi(\rho) \leq p, \\ V_T(\rho) < V_L(\rho) & \text{implies } \pi(\rho) = \rho p + (1-\rho)(1-p); \end{cases}
$$

(ii) $\quad W(\rho) = \text{Max}\ [V_T(\rho), V_L(\rho)]$.

We shall in fact concentrate on symmetric equilibria associated with a value function W of the opportunistic journalist which is continuous and nondecreasing in his reputation ρ. To study the existence of such an equilibrium, we proceed in two steps. First, we take as given a continuation value function W and solve for the equilibrium of the associated one-shot game. Using the continuity and

monotonicity properties of the resulting payoff, we then find the equilibrium function W as the fixed point of a contraction mapping. Let C_+ be the space of continuous nondecreasing functions from [0,1] into R, endowed with the sup norm (for any W_1, W_2 in C_+, $||W_1 - W_2|| = \sup_{\rho \in [0,1]} (|W_1(\rho) - W_2(\rho)|)$. The main properties of the equilibrium are derived from the following lemma:

Lemma 4.4: *Consider a function* W *in* C_+. *For any* ρ in [0,1], *there is a unique* $\pi^*(\rho,W)$ (*and associated* $V_T^*(\rho,W)$, $V_L^*(\rho,W)$ *given by* (4.10)-(4.11)) *which satisfies* (4.13). π^* *is a continuous function from* $[0,1] \times C_+$ *into* [1/2,p] *and*:

$$(4.14) \qquad \begin{cases} \pi^*(\rho,W) < p & \text{for all } \rho < 1 \\ \pi^*(1,W) = p \end{cases}$$

The functions $V_T^*(\rho,W)$ and $V_L^*(\rho,W)$ *from* $[0,1] \times C_+$ *into* R *are continuous and nondecreasing in their two arguments.*

Lemma 4.4 says that for any continuation value W of the reputation at the end of the period, there is a unique well-defined equilibrium of the current period game. The resulting intertemporal utility of the opportunistic journalist increases, whether he lies or he tells the truth, with his reputation ρ and with the end of period valuation W.

Let now T(.) be the mapping associating to each W in C_+ the function $T(W): \rho \in [0,1] \to \mathrm{Max}[V_T^*(\rho,W), V_L^*(\rho,W)] \in R$. From Lemma 4.4, it is easy to show, using the standard argument of Blackwell [1965], that T is a contraction on C_+; hence:

<u>Theorem 4.5</u>: <u>There is a unique symmetric equilibrium of the repeated information transmission game, associated with a continuous and nondecreasing value function</u> W.

In equilibrium, information, accurate or false, is transmitted and acted upon: $\pi(\rho) = \pi^*(\rho, W(\rho)) \in (1/2, p)$ by Lemma 4.4. This allows the opportunistic journalist to take advantage of his private information and of the public's imperfect knowledge of his objectives to manipulate the market price through misleading announcements.

We were not able to rule out the existence of nonsymmetric equilibria as we did in the single-period game. Nor could we exclude equilibria associated to a discontinuous or non-monotone function W; these, however, (if any) do not seem very natural. Indeed, a proof by induction using Lemma 4.4 shows that when the game is played a finite number K of times, it has a unique symmetric equilibrium associated to a value function $W_K \in C_+$. If the infinite game's solution is to be considered in any sense as the limit of the finite game's solution as K tends to infinity, W will be the limit of the W_K's, and therefore also an element of C_+.

4c. Equilibrium strategies and credibility

We now examine how the journalist's strategy $q(\rho)$ and credibility $\pi(\rho)$ vary with the state of his reputation ρ.

For good or (very) bad reputations, it is possible to characterize them precisely. Note from (4.9) or (4.12) that, in equilibrium, the revised reputation is much less sensitive to whether or not the prediction is confirmed when ρ is near zero or one than when it is well inside (0,1). The equilibrium incentive to be honest is

therefore not monotonic, and in particular: $W(\rho^{+}_{t+1}) - W(\rho^{-}_{t+1}) = 0$ at $\rho = 0$ and at $\rho = 1$. Near these extreme points, reputation effects thus become unimportant, and one should expect an outcome similar to that of the static game. Indeed:

Proposition 4.6: The functions $q(\rho)$ and $\pi(\rho)$ are continuous on $[0,1]$, and $q(0) = \pi(0) = 1/2$. Moreover, there exists $\bar{\rho} \in (0,1)$ such that, for all ρ in $[\bar{\rho},1]$: $q(\rho) = 0$ and $\pi(\rho) = p\,\rho + (1-p)\,(1-\rho)$.

The opportunistic journalist always lies when his reputation is above a certain threshold $\bar{\rho}$: he runs it down, in expected terms. Conversely, when it is very low, he does not find it worthwile to make a significant effort to rebuild it (when $\rho \approx 0$, $q \approx 1/2$), since only a long (unprofitable) sequence of correct forecasts could convince the public to pay attention to his messages again.

Note that for $\rho > \bar{\rho}$, π increases in ρ. The dependence of π on ρ in the range where $V_T = V_L$ is more complicated. Given π, a higher ρ raises both ρ^{+}_{t+1} and ρ^{-}_{t+1}; as seen from (4.12), the net effect on the incentive to tell the truth has the sign of $(p/\pi)W'(\rho p/\pi) - ((1-p)/(1-\pi))W'(((1-p)\rho/(1-\pi)))$. In regions where W has a coefficient of relative risk-aversion $-\rho.W''(\rho)/W'(\rho)$ smaller than 1 (so that $\rho.W'(\rho)$ increases), this effect is positive, and π will increase; where $-\rho.W''(\rho)/W'(\rho) < 1$, however, the effect will be negative, so that $\pi(\rho)$ could decrease. Since we know very little about the value function $W(\rho)$, the sign of the variations of π with ρ remains ambiguous.[10]

The same indeterminacy holds for the journalist's strategy

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[10] For instance, in the two stage game with quadratic utility functions (leading to: $U(1,\pi) = 1 - 4p + 8p\pi - 4\pi^2$), one finds an interval $[\rho_1, 1/2]$ where π decreases with ρ.

$q(\rho)$. The first effect of an increase in ρ is to raise π (see 4.8), thereby increasing the journalist's profit-related incentive to lie, i.e. to lower q. Thus π may increase while q decreases. On the other hand, if π ever decreases in ρ, so does q by (4.8).

Given these problems, we are not able to determine whether or not intermediate reputations are worth improving by "investing in truth", i.e. whether q rises above 1/2 before decreasing to zero. However, the extension of the model to the case where the journalist's payoff depends on an i.i.d. random variable $\tilde{\theta}$, as in Sobel [1985] (see Section 6a.) is sufficient to generate such behavior: when ρ is not too low but the payoff to dishonesty in state θ is very small, it is worth to try and upgrade the reputation while waiting for better days.

While we cannot characterize the variations of $\pi(\rho)$ over [0,1], we can establish a more indirect property of this function:

<u>Proposition 4.7</u>: $\dfrac{p\rho}{\pi(\rho)}$ <u>and</u> $\dfrac{(1-p)\rho}{1-\pi(\rho)}$ <u>are increasing functions of</u> ρ.

The journalist's end-of-period equilibrium reputation thus always increases with ρ, whether his forecast turns out to be correct or wrong.

5. <u>The survival of gurus and newsletters: reputation in the long run.</u>

5.a <u>Constant type</u>

Casual empiricism leads to the impression that most individuals' influence on the market does not last very long. The majority of gurus are "shooting stars" who rapidly fade into oblivion, often after having made a series of bad calls or being linked to some insider-trading or market-manipulating scheme. We examine this issue in the light of our model by analyzing the stochastic properties and long-run behavior of the journalists' reputation along the equilibrium path.

We want to know in particular whether the opportunistic journalist can (profitably) hide his type indefinitely.

The reputation $(\rho_t)_{t\epsilon N}$ follows a Markov process (see 4.9) which depends in a complicated way on the signals $(\tilde{s}_t)_{t\epsilon N}$ received by the journalist and the publicly observed returns $(\tilde{n}_t)_{t\epsilon N}$, on the equilibrium signalling strategy q(.) -or equivalently on the belief function $\pi(.)$- and finally on the actual type of the journalist.

Consider the probabilities associated to the two possible transitions from ρ_t, i.e. to ρ^+_{t+1} and ρ^-_{t+1}. For the public, which does not know the journalist's type, $\text{Prob}[\tilde{n}_t = \tilde{m}_t | \Omega_t] = \pi_t$, so ρ_t is a martingale, as in (4.5). However, when the journalist is in fact honest (although the public does not know it), $\text{Prob}[\tilde{n}_t = \tilde{m}_t | H] = p \geq \pi_t$, so:

$$(5.1) \qquad E[\rho_{t+1} | \Omega_t, H] = \rho_t \left[\frac{p^2}{\pi_t} + \frac{(1-p)^2}{1-\pi_t} \right] \geq \rho_t$$

with <u>strict</u> inequality when ρ_t is different from 0 or 1. In other words, when the journalist is honest, ρ_{t+1} is a (strict) <u>submartingale</u>. It therefore converges almost surely to a stationary random variable, which we can show to be equal to one (almost surely). Similarly, the fact that the dishonest journalist always lies to some degree ($q_t < 1$, or $\pi_t < p$) implies that on average his reputation will deteriorate: $\text{Prob}[\tilde{n}_t = \tilde{m}_t | O] = pq_t + (1-p)(1-q_t) = (\pi_t - \rho_t p)/(1-\rho_t) \leq \pi_t$, so that ρ_t is a supermartingale:

$$(5.2) \qquad E[\rho_{t+1} | \Omega_t, O] \leq \rho_t$$

with <u>strict</u> inequality for $\rho_t \in (0,1)$, which can be shown to converge to

zero almost surely. We therefore prove:

Theorem 5.1: *Along the equilibrium path, (for any ρ_1 in (0,1)), the reputation process $\{\rho_t\}_{t\in N}$ converges almost surely to 0 (resp. to 1) as t goes to infinity if the journalist is opportunistic (resp. honest).*

The journalist thus reveals his type in the long run, and this will affect market volatility. As an opportunistic journalist gradually loses his reputation, the market price becomes less responsive to his announcements: when ρ_t goes to zero, π_t goes to 1/2 by Proposition 4.6, and $\tilde{P}(\pi_t)$ converges to $\tilde{P}(1/2)$. Conversely, as an honest journalist solidifies his reputation, price volatility tends to increase over time, since $\tilde{P}(\pi_t)$ converges to $\tilde{P}(\rho)$.

In the meantime, when the journalist is dishonest, those who are repeatedly deceived and lose money are the rational speculators, whereas "noise traders" generally make money along with the manipulator; the converse holds true when the journalist is honest.

5b. Changing types

The above result of complete asymptotic learning by the public rests on the maintained asssumption that the journalist's type is determined once and for all at the beginning of the game. The model can be generalized to the case where his type $\tilde{J}_t \in \{H,O\}$ evolves according to a Markov process:

$$(5.3) \qquad \begin{cases} \text{Prob}[\tilde{J}_{t+1} = H | \tilde{J}_t = H] = \phi \\ \text{Prob}\ [\tilde{J}_{t+1} = O | \tilde{J}_t = O] = \psi \end{cases}$$

We will assume that there is some (imperfect) persistence in the journalist's type, in the following sense:

$$\phi,\ \psi \text{ are in } (0,1) \text{ and } \phi + \psi > 1.$$

The stochastically changing type of a given journalist will represent here a fluctuating ability (privately known) to trade without being detected, in a context where he is forbidden to do so. Thus, when secret trades are not possible, his instantaneous payoff is $U^H(s,\pi) \equiv u_0(w_0)$, leading him to act honestly; when they are, it is $U^O(s,\pi) = U(s,\pi)$ given by (2.3). Alternatively, different journalists (resp. executives) could be writing the financial column (resp. running the company whose stock is traded) in different periods. One could think for instance of an imperfect monitoring and screening process, subject to both type I and type II errors, which is used to detect and replace (potentially) dishonest agents.

Let Ω_t' denote the public's information at the end of period t; in addition to Ω_t, it includes the realizations of $\tilde{m}_t$ and $\tilde{n}_t$. The updating formula is now, in both cases:

$$\begin{aligned}\text{Prob}[\tilde{J}_{t+1}=H|\Omega_{t+1}] = {} & \text{Prob}[\tilde{J}_t=H|\Omega_t'].\text{Prob}[\tilde{J}_{t+1}=H|\tilde{J}_t=H] \\ & + \text{Prob}[\tilde{J}_t=0|\Omega_t'].\text{Prob}[\tilde{J}_{t+1}=H|\tilde{J}_t=0],\end{aligned}$$

so:

$$(5.4) \qquad \rho_{t+1} = \begin{cases} \rho_{t+1}^{+} = \dfrac{p\rho_t}{\pi_t}\,\phi + \left[1 - \dfrac{p\rho_t}{\pi_t}\right](1-\psi) & \text{if } m_t = n_t \\[2ex] \rho_{t+1}^{-} = \dfrac{(1-p)\rho_t}{1-\pi_t}\,\phi + \left[1 - \dfrac{(1-p)\rho_t}{1-\pi_t}\right](1-\psi) & \text{if } m_t \neq n_t \end{cases}$$

Note that ρ_{t+1}^{+} (resp. ρ_{t+1}^{-}) is increasing in ρ_t/π_t (resp. in $\rho_t/(1-\pi_t)$), due to the persistence in the journalist's type ($\phi + \psi > 1$).

The dynamic programming problem of the opportunistic journalist is different in the case of uncertain survival and in that of uncertain preferences,[11] but the fundamental properties of the equilibrium remain unchanged from the previous sections. Indeed, given that the above monotonicities in the values for the updated reputation are preserved, the construction of Section 4b can be replicated, showing the existence and uniqueness of a symmetric equilibrium with increasing valuation function(s).

The stochastic properties of the equilibrium are described by the joint process $(\tilde{J}_t, \tilde{\rho}_t)$ governed by (5-3)-(5-4). Standard compactness and continuity arguments show the existence of an invariant distribution, but we can only (strongly) conjecture that $(\tilde{J}_t, \tilde{\rho}_t)$ converges in the long run to a stationary random variable $(\tilde{J}^*, \tilde{\rho}^*)$ characterized by such a distribution.[12] The evolution of $\tilde{J}_t$ is autonomously determined by (5.4), and in the long run it does converge to a unique stationary random variable $\tilde{J}^*$, with:

$$(5.5) \qquad \text{Prob}[\tilde{J}^* = H] = \frac{1-\psi}{2-\phi-\psi} \equiv \rho^*, \quad \text{Prob}[\tilde{J}^* = 0] = \frac{1-\phi}{2-\phi-\psi} \equiv 1-\rho^*$$

As to the limiting distribution $\tilde{\rho}^*$, it is clear from (5.4), that it can not be degenerate, so that random renewal prevents the public from ever fully discovering the privately informed agent's current type (as in Holmström [1983] and Cuckierman-Meltzer [1986]). As a result, his

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11 In both cases the new values of ρ^+_{t+1} and ρ^-_{t+1} as functions of ρ must be substituted into $V_L(\rho)$ and $V_T(\rho)$; in the first case, β is then simply replaced by $\beta\psi$; in the second one, value functions $W^H(\rho)$ and $W^O(\rho)$ must be defined, leading to four equations in place of (4.10)-(4.11), and to a fixed point on the couple (W^H, W^L).

12 We do not know of any applicable theorem.

opportunities for manipulation (which he takes advantage of when he is dishonest, i.e., in a fraction ρ^* of periods) do not vanish over time as in Section 5b, but remain stationary. Whereas individual newsletters or gurus may be short-lived, the phenomenon may well endure and flourish indefinitely.

6. Extensions of the basic model

6a. State dependent payoffs

Our extension of Sobel's [1985] results to games with an imperfectly informed sender carries over to the case where the latter's payoff in each period depends on some state variable $\tilde{\theta}_t$, where the $\tilde{\theta}_t$'s are independently and publicly drawn from a common distribution at the beginning of period t. It suffices to add $\tilde{\theta}$ as an argument in strategies, beliefs, utility and value functions. In particular, if $\tilde{\theta}$ has a monotonic effect on the incentive to lie, i.e.:

$$(6.1) \qquad s.\frac{\partial^2 U(s,\pi;\theta)}{\partial\theta\ \partial\pi} < 0$$

then the equilibrium strategy $q(\rho;\theta)$ and credibility $\pi(\rho;\theta)$ will be non-increasing functions of the current value of $\tilde{\theta}$. In our asset market model, intuition suggests that an increase in the number α of agents who pay attention to the journalist's message will increase his control of the market price, hence also his incentive to lie. Because changes in α also induce wealth effects through their impact on the equilibrium price distribution, we can only verify this intuition under some very restrictive assumptions:

Proposition 6.1: If agents' utility functions are logarithmic and $\alpha \le 1/2$, α satisfies condition (6.1). Therefore, if the number of

rational speculators in each period is an i.i.d. process $\{\alpha_t\}_{t \in N}$, then: $\partial q(\rho;\alpha)/\partial\alpha \le 0$ and $\partial\pi(\rho;\alpha)/\partial\alpha \le 0$ for all ρ in $(0,1)$ and α in $[0,1/2]$.

In this case, the more people are reached by the journalist's forecast (or the more reknowned the guru) the less confident they are about its reliability.

Similarly, assume that a journalist who lies risks a penalty of expected value $\tilde{K}$, which is to be substracted from his utility $U(\tilde{s},\tilde{\pi})$. $\tilde{K}$ reflects the authorities' monitoring ability and toughness, but also depends on the individual's ability to hide. An honest journalist is one for which $\tilde{K}^H > U(1,1-p) - U(1,p)$ with probality one. An opportunistic journalist is one for which the $\tilde{K}^O_t$'s are (publicly observable) i.i.d. random variables, violating the preceding inequality with positive probability. Using (4.12), it is straightforward to show that the opportunistic journalist' strategy $q(\rho;K)$ and the public credibility $\pi(\rho;K)$ of a journalist of unknown type are non-decreasing functions of the current value of the penalty $\tilde{K}^O$.

6b. Combining the two types of market manipulation.

In Section 2c we identified and gave examples of two market manipulation schemes, corresponding to the privately informed agent's trading before or after his announcement. Until now, we have focused on the second one, which yields a more tractable problem and is more directly related to informational considerations. Both types of schemes, however, are potentially important in actual markets, and their combination is particularly attractive (see Figure 2); we therefore examine now how the model changes when Assumption A4 is lifted.

In the first subperiod, public trading is based solely on the prior $\pi = 1/2$, and simply results in an uninformative price $P_0 = 1/2$, with (almost) all traders $a \in (0,1]$ keeping their initial holdings of zero.[13] The market-clearing price $\tilde{P}(\pi)$ at $\tau = 1/2$ therefore still satisfies Properties 2.1 and 2.3.

As to the journalist, he now faces a two-stage decision problem (see Figure 2). If B_1 and B_2 denote his holdings of the asset in the first and second subperiods, his wealth will be $w_0 + (\tilde{P}-1/2)B_1$ at $\tau = 1/2$, and $w_0 + (\tilde{P}-1/2)B_1 + (\tilde{n}-\tilde{P})B_2$ at $\tau = 1$. To prevent riskless speculation in the first subperiod, both $\tilde{P} > 1/2$ and $\tilde{P} < 1/2$ must have positive probability, i.e. 1/2 must lie inside the support of M_π, for all π. This requires that α not be too large, compared to the dispersion in the distribution of $\tilde{\pi}$; thus noise traders, whose role with respect to post-announcement trading is mostly incidental, are crucial to generate riskiness in pre-announcement trades; we shall come back to this point later.

If the observed signal is s and the planned message m induces beliefs π, the optimal combination of pre-announcement and post-announcement speculation by the journalist consists of trades $B_1^{*s}(\pi)$ at $\tau = 0$ and trade plans $B_2^{*s}(w_0+(P-1/2)B_1^{*s},P)$ for all P at $\tau = 1/2$, which maximize his expected payoff as of $\tau = 0$:

$$(6.2)\qquad U^*(s,\pi) = \max_{B_1}\Big[\int_{1-p}^{p} \max_{B_2} \{p^s\, u_0(w_0 + (P-1/2)B_1 + (1-P)B_2)$$
$$+ (1-p^s)\, u_0(w_0 + (P-1/2)B_1 - PB_2)\}\, dM_\pi(P)\Big].$$

---

[13] Recall that we restrict the noise to the second subperiod, since we are primarily interested in the interaction of its innovation between $\tau = 0$ and $\tau = 1/2$ with the journalists' message.

The additional complexity comes from the fact that the journalist can now also speculate on his sole power to influence the market <u>irrespectively</u> of his information. Whereas the profitability of post-announcement speculation rests on his ability to induce erroneous beliefs in the public (making $s\pi$ as small as possible), that of pre-announcement speculation is determined by the degree to which he can affect beliefs--i.e. destabilize the price--in <u>any</u> direction (making $|\pi-1/2|$ as large as possible, whatever the value of s).[14]

Because it embodies both of the above effects, the privately informed agent's expected utility $U^*(s,\pi)$ at time zero (when he decides what message to send at $\tau = 1/2$ and chooses $B_1^{*s}(\pi)$ accordingly) may not satisfy Property 2.2 any more. Since this monotonicity condition was key to the proof of existence and uniqueness of the equilibrium, we must examine under which conditions it still holds in the present case. Given symmetry, let $\tilde{s} = 1$; we shall abbreviate $B_1^{*+}(\pi)$ as B_1^*, and $B_2^{*+}(w_0+(P-1/2)B_1^*, P)$ as B_2^{**}. The necessary and sufficient condition for Property 2.2 to hold (derived in Section 7) is:

$$(6.3) \qquad \int_{1-p}^{p} [p\, u_0'(w_0 + (P-1/2)B_1^* + (1-P)B_2^{**}) + (1-p)\, u_0'(w_0 + (P-1/2)B_1^* - PB_2^{**})](B_2^{**} - B_1^*)\, \frac{dM_\pi}{d\pi}(P)\, dP < 0$$

Since $dM_\pi/d\pi \le 0$, it will hold if the journalist prefers to trade more, in an appropriately weighted average sense, in the second subperiod than

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[14] This difference is clearly apparent in the two examples of Section 2c. Nathan Rothschild had a strong incentive to announce the opposite of what he knew about the battle of Waterloo, then trade. The scheme in the Rex vs. de Berenger case, on the hand, could have worked just as well by selling British securities short before spreading rumors of a major victory by Napoleon as it did by buying them before spreading rumors of his death.

in the first, i.e. if the solution B_1^* to (6.2) is small enough. Indeed, the realizations of $\tilde{P}$ which are large enough to have B_1^* (significantly) greater that $B_2^{**} = B_2^*(w_0+(\tilde{P}-1/2)B_1^*,\tilde{P})$, or:

$$(6.4) \qquad \frac{\tilde{P}}{1-\tilde{P}} > \frac{p}{1-p} \cdot \frac{u_0'(w_0 + B_1^*/2)}{u_0'(w_0 - B_1^*/2)}$$

must have a low enough probability under M_π. The system (6.2)-(6.3) is generally too complicated to derive an explicit condition on the underlying parameters, but it suggests that monotonicity will obtain if speculation is "safer" in the second subperiod than in the first, i.e., if the journalist's information on the final value of the asset is quite reliable (p large enough) while his ability to move the intermediate price is limited, due to noise traders (α small enough).

This intuition can be formalized when agents' utility functions are logarithmic. In this case, the journalist's expected utility from his optimal two-stage speculation scheme separates into a pre-announcement and a post-announcement component (see Section 7):

$$(6.5) \qquad U^*(1,\pi) = \int_{1-p}^{p} \mathrm{Log}(w_0 + (P-1/2)B_1^*)\, dM_\pi(P)$$
$$+ \int_{1-p}^{p} [p\, \mathrm{Log}(p/P) + (1-p)\, \mathrm{Log}((1-p)/(1-P))]\, dM_\pi(P).$$

As expected, the second term decreases with π (the integrand is a decreasing function of P, and $\tilde{P}(\pi)$ increases stochastically in π), while the first term increases in π for $B_1^* > 0$, which corresponds to $\pi > 1/2$ (as long as noise traders' beliefs are unbiased on average, so that $M_{1/2}$ is symmetric around 1/2) and decreases in π for $B_1^* < 0$,

which corresponds to $\pi < 1/2$. Assume for instance that $\tilde{\pi}$ is uniformly distributed on $[1-p,p]$, so that $\tilde{P}(\pi) = \alpha\pi + (1-\alpha)\tilde{\pi}$ is uniformly distributed on $[\underline{P}(\pi), \overline{P}(\pi)] = [\alpha\pi + (1-\alpha)(1-p), \alpha\pi + (1-\alpha)p]$. Assume also that $\alpha < 1/2$, which ensures that $1/2 \in (\underline{P}(\pi), \overline{P}(\pi))$ for all π, so that both speculative schemes are risky. It can then be shown that $B_1^* = 0$ if $\alpha = 0$,[15] and that, for small values of α:

$$B_1^* \approx 3\alpha \frac{\pi - 1/2}{(p-1/2)^2} w_0 \tag{6.6}$$

which increases in α and decreases in p. As a result, we have:

<u>Proposition 6.2</u>: <u>If agents' utility functions are logarithmic and</u> $\tilde{\pi}$ <u>is distributed uniformly on</u> $[1-p,p]$, <u>there exists</u> $\bar{\alpha} \in (0,1/2)$ <u>such that, for all</u> $\alpha \le \bar{\alpha}$, $U^*(s,\pi)$ <u>satisfies Property 2.2, hence all previous results remain valid in the presence of both types of speculation.</u>

6c. <u>Determining ability as well as honesty</u>

Realistically, the public's faith in the predictions of a journalit or a "guru" depends as much on their assessment of the accuracy of his private information as on their confidence in his truthfully reporting it. We have been treating the accuracy of the journalist's signal ($p > 1/2$) as both exogenously given and publicly known; relaxing these assumptions would generate two interesting extensions of the model.

First, one could let p be either 1/2 (ignorant journalist) or $p = \bar{p} > 1/2$ (competent journalist) and consider a similar game with

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[15] This property does not hold for arbitrary utility functions.

two reputation state variables, $\rho_t = \text{Prob}[H|\Omega_t]$, $\nu_t = \text{Prob}[p=\bar{p}|\Omega_t]$. The updating and dynamic programming formulas are very similar to those of Section 4, but the cross-effects between ρ and ν substantially complicate their resolution.

Secondly, one could examine the journalist's incentive to acquire private information, possibly at some cost. As in Cuckierman and Meltzer [1986], the opportunistic journalist does not want p to be too close to one half (obviously) nor to one, because $p < 1$ allows him to mislead the market repeatedly instead of just once, as in Sobel [1985]. If he publicly chooses p after discovering his type, and types are permanent, the problem becomes a signalling game. More interestingly, if he chooses p before learning his type, or if his type changes over time as in Section 5b, there will be an optimal level of accuracy which balances the benefits of superior information with those of ambiguity about the true cause of erroneous predictions.

Conclusion

We showed how a financial journalist, an insider or a "guru" can repeatedly manipulate an asset market to his own benefit, by making strategically distorted announenements or predictions. We also analyzed the extent to which the public's attempt to learn over time whether this informed agent can be trusted or not may limit, in the long run, his influence on the market.

While our results may provide realistic explanations for some stylized facts of actual asset markets, it should be kept in mind that they were derived in the context of a very specific model, often under

rather restrictive assumptions. In particular, what happens when the agent can set a price for his information or when there are more than two states of nature remains an open question. Nonetheless, our results suggest that a concern for the veracity of the informations communicated to markets, and embodied in prices, may justify placing some restrictions on trades (since speech itself is unrestricted) by insiders.

7. Proofs

Proof of Property 2.1:

Given a price P in (0,1), the asset demand of agent a is determined by maximizing:

$$\Psi_a(B: P,\pi_a) = \pi_a\, u_a(w_a+(1-P)B) + (1-\pi_a)u_a(w_a-PB) \tag{7.1}$$

with respect to B in R. By assumption A1, Ψ_a is continuously differentiable in all its arguments and strictly concave in B. Moreover, for all (P,π_a) in $(0,1)^2$:

$$\lim_{B\to\infty} \partial\Psi_a(B: P,\pi_a)/\partial B = -P(1-\pi_a)\, u'(-\infty) = -\infty$$
$$\lim_{B\to\infty} \partial\Psi_a(B; P,\pi_a)/\partial B = (1-P)\, \pi_a\, u'(-\infty) = +\infty.$$

so that the unique solution $B_a(P,\pi_a)$ of agent a's optimization problem is determined by the first order condition:

$$\frac{u_a'(w_0 + (1-P)B_a)}{u_a'(w_0 - PB_a)} = \frac{1-\pi_a}{\pi_a} \cdot \frac{P}{1-P} \tag{7.2}$$

Note that $B_a > 0$ (resp. < 0) as $P < \pi_a$ (resp. $P > \pi_a$). By the implicit function theorem, $B_a(.,.)$ is continuously differentiable and $\partial B_a(P,\pi_a)/\partial P$ has the sign of:

(7.3) $\partial^2\Psi_a(B_a;\ P,\pi_a)/\partial P\partial B = -\ \pi_a\ u_a'(w_a+(1-P)B_a)\ -\ (1-\pi_a)u_a'(w_a-PB_a)$

$+\ B_a\ \{-\pi_a(1-P)\ u_a''(w_a+(1-P)B_a)\ +\ (1-\pi_a)\ P\ u_a''(w_a-PB_a)\}.$

The first two terms are negative. To show that the third one is nonpositive, assume first that $P \le \pi_a$ so that $B_a \ge 0$; by assumption A1:

$$-\frac{u_a''(w_a+(1-P)B_a)}{u_a'(w_a+(1-P)B_a)} \le -\frac{u_a''(w_a-PB_a)}{u_a'(w_a-PB_a)}$$

so that the term multiplying B_a in (7.3) is no greater than:

$$\frac{u_a''(w_a-PB_a)}{u_a'(w_a-PB_a)}\ [(1-\pi_a)P\ u_a'(w_a-PB_a)\ -\ \pi_a(1-P)\ u_a'(w_a+(1-P)B_a)] = 0$$

by (7.2). A similar reasoning applies when $P > \pi_a$ and $B_a < 0$, so that $B_a(P,\pi_a)$ always decreases in P.Similarly, $\partial B_a(P,\pi_a)/\partial\pi_a$ has the sign of:

$$\partial^2\Psi_a(B_a;\ P,\pi_a)/\partial\pi_a\partial B = (1-P)\ u_a'(w_a+(1-P)B) + P\ u_a'(w_a-PB) > 0;$$

so:

(7.4) $\partial B_a(P,\pi_a)/\partial P < 0;\quad \partial B_a(P,\pi_a)/\partial\pi_a > 0.$

Given the beliefs π and $\tilde{\pi}$ of rational and noise traders, and the fact that agent zero's trades have a negligible impact on the market, a price P clears the market if:

(7.5) $\xi(P,\pi,\tilde{\pi}) \equiv \int_0^{\alpha} B_a(P,\pi)\ da + \int_{\alpha}^{1} B_a(P,\tilde{\pi})\ da = 0.$

Note that, by A1, the B_a's are continuous, hence summable, in a. By (7.4), $\xi(P,\pi,\tilde{\pi})$ decreases in P. If $P < 1-p$, then $P < \pi$ and $P < \tilde{\pi}$, hence $B_a(P,\pi) > 0$ for all a in $(0,\alpha]$ and $B_a(P,\tilde{\pi}) > 0$ for all a in

$(\alpha,1]$, so that $\xi(P,\pi,\tilde{\pi}) > 0$. Similarly, if $P > p$, $\xi(P,\pi,\tilde{\pi}) < 0$. Therefore (7.5) has a unique solution $P^*(\pi,\tilde{\pi})$ in $[1-p,p]$. Finally by (7.4): $\partial\xi(P,\pi,\tilde{\pi})/\partial\pi > 0$, so that by the implicit function theorem:

$$(7.6) \qquad \partial P^*(\pi,\tilde{\pi})/\partial\pi > 0 \text{ for all } \pi \text{ and } \tilde{\pi} \text{ in } [1-p,p].$$

The random variable $\tilde{P}(\pi) = P^*(\pi,\tilde{\pi})$ is therefore increasing (in the sense of first order stochastic dominance) in π, so that $M_\pi(.)$ is decreasing. Q.E.D.

Proof of Property 2.2:

At $\tau = 1/2$, the price is $P = P^*(\pi,\tilde{\pi})$ and the journalists' demand is $B^s(P) \equiv B_0(P,p^s)$, given by (7.2) with $\pi_0 = p^s$. Note that $s.B^s(P) > 0$. Coming back to $\tau = 0$, his expected utility is by (2.3):

$$(7.7) \qquad U(s,\pi) = \int_{1-p}^{p} \hat{U}^s(P^*(\pi,\tilde{\pi}))\, dL(\tilde{\pi}),$$

where $\hat{U}^s(P) \equiv p^s\, u_0(w_0+(1-P)B^s(P)) + (1-p^s)\, u_0(w_0-PB^s(P))$ for all P in $[1-p,p]$, and dL denotes the distribution of $\tilde{\pi}$. For all $\tilde{\pi}$, the function $P^*(.,\tilde{\pi})$ is differentiable and increasing (see (7.6)); moreover, $\hat{U}^s(.)$ is also differentiable, and:

$$\hat{U}^{s\prime}(P) = -[p^s\, u_0'(w_0+(1-P)B^s(P)) + (1-p^s)\, u_0'(w_0-PB^s(P))]\, B^s(P)$$

which has the sign of $-B^s(P)$, i.e. of $-s$, hence Property 2.2. Q.E.D.

For the sequel it will be useful to rewrite (7.7) as:

$$(7.8) \qquad U(s,\pi) = \int_{1-p}^{p} [p^s\, u_0(w_0 + (1-P)B^s(P)) + (1-p^s)\, u_0(w_0 - PB^s(P))]\, dM_\pi(P).$$

Proof of Property 2.3:

Equation (7.8) implies:

$$
\begin{aligned}
U(-1,\pi) &= \int_{1-p}^{p} \max_{B} \{p\ u_0(w_0-PB) + (1-p)\ u_0(w_0+(1-P)B)\}\ dM_\pi(P) \\
&= \int_{1-p}^{p} \max_{B} \{p\ u_0(w_0-(1-P')B) + (1-p)\ u_0(w_0+P'B)\}\ dM_\pi(1-P') \\
&= \int_{1-p}^{p} \max_{B} \{p\ u_0(w_0+(1-P')B') + (1-p)\ u_0(w_0-P'B')\} dM_{1-\pi}(P')
\end{aligned}
$$

which, by assumption A4, is equal to $U(1,1-\pi)$. Q.E.D.

Proof of Proposition 3.1

1) Let $\rho \leq 1/2$, and assume that $\pi^+ > \pi^-$. By Property 2.2, $U(\bar{s},\pi^+) > U(\bar{s},\pi^-)$ (resp. <) if and only if $\bar{s} = -1$ (resp. $\bar{s} = +1$). Definition 3.2 then implies:

$$
(7.9) \quad \begin{cases} \pi^+ = \rho\ p + (1-\rho)\ \mathrm{Prob}[\tilde{n}=1|\bar{s}=-1] = \rho\ p + (1-\rho)\ (1-p) \\ \pi^- = \rho\ (1-p) + (1-\rho)\ \mathrm{Prob}[\tilde{n}=1|\bar{s}=1] = \rho\ (1-p) + (1-\rho)\ p \end{cases}
$$

Thus, since $p > 1/2 \geq \rho$: $\pi^+ \leq \pi^-$, a contradiction. Similarly, one can exclude $\pi^+ < \pi^-$. Let us now show that $\pi^+ = \pi^- = 1/2$ defines an (uninformative) equilibrium. Whatever his message, the sender gets $U(\bar{s},1/2)$. Being indifferent, he can randomize between telling the truth and lying, with probabilities q^+ (when $\bar{s} = 1$) and q^- (when $\bar{s} = -1$) which solve the equilibrium conditions (ii) in Definition 3.1:

$$
1/2 = \frac{\rho p + (1-\rho)[q^+ p + (1-q^-)(1-p)]}{\rho + (1-\rho)(q^+ + 1 - q^-)} = \frac{\rho(1-p) + (1-\rho)[(1-p)q^- + p(1-q^+)]}{\rho + (1-\rho)(q^- + 1 - q^+)}
$$

which are equivalent (as $\rho < 1$) to $q^+ = q^- = (1-2\rho)/(2(1-\rho)) \equiv q^*$. Finally, there can be no equilibrium with $\pi^+ = \pi^- \neq 1/2$, because $\pi^+ > 1/2$ (say) requires $q^+ + q^- > 2q^*$ while $\pi^- > 1/2$ requires $q^+ + q^- < 2q^*$.

2) Now let $\rho > 1/2$. As above, if $\pi^+ > \pi^-$, (π^+,π^-) must be given by (7.9) (the dishonest sender always lies: $q^+ = q^- = 0$. Conversely, this couple of values satisfies all the conditions of Definition 3.1. There is no other equilibrium, because, as above $\pi^+ < \pi^-$ and (7.9) imply $\pi^+ = p > 1-p = \pi^-$, a contradiction, while $\pi^+ = \pi^-$ requires $q^+ + q^- = (1-2\rho)/(1-\rho) < 0$, another impossibility. Q.E.D.

Proofs for Section 4:

We now turn to the study of the infinite game, and start with a preliminary lemma. For π in $[1-p,p]$, ρ in $[0,1]$ and W in C_+, define:

$$(7.10) \qquad V_{0T}(\pi;\ \rho,W) = U(1,\pi) + \beta p\ W\left[\frac{p\rho}{\pi}\right] + \beta(1-p)\ W\left[\frac{(1-p)\rho}{1-\pi}\right]$$

$$(7.11) \qquad V_{0L}(\pi;\ \rho,W) = U(1,1-\pi) + \beta(1-p)\ W\left[\frac{p\rho}{\pi}\right] + \beta p\ W\left[\frac{(1-p)\rho}{1-\pi}\right]$$

$$(7.12) \qquad F(\pi;\rho,W) = V_{0T}(\pi;\rho,W) - V_{0L}(\pi;\ \rho,W)$$

$$= U(1,\pi) - U(1,1-\pi) + \beta(2p-1)\left[\ W\left[\frac{p\rho}{\pi}\right] - W\left[\frac{(1-p)\rho}{1-\pi}\right]\ \right]$$

The following lemma is the key to the proof of Lemma 4.4:

Lemma 7.1: For any ρ in $[0,1]$ and W in C_+, $F(.;\rho,W)$ is strictly decreasing on $[0,1]$. Moreover: $F(1/2;\ \rho,W) \geq 0$; $F(p;\rho,W) < 0$. The equation $F(\pi;\ \rho,W) = 0$ therefore has a unique root $\pi_1(\rho,W)$, which maps $[0,1] \times C_+$ into $[1/2,p)$ continuously. The function $V_1(\rho,W)$, defined on $[0,1] \times C_+$, by:

$$V_1(\rho,W) = V_{0T}(\pi_1(\rho,W);\ \rho,W)$$

is nondecreasing in each of its two arguments.

Proof: By Property 2.2, $U(1,\pi) - U(1,1-\pi)$ is decreasing in π. Since W is in C_+, $W(p\rho/\pi) - W((1-p)\rho/(1-\pi))$ is non-increasing in π. Therefore $F(\pi;\ \rho,W)$ is decreasing in π; it is also clearly continuous in all its arguments.

Since $p > 1/2$ and $W \in C_+$, $F(1/2;\ \rho,W) \geq 0$. From Property (2.2), $U(1,p) - U(1,1-p) < 0$, and consequently $F(p;\ \rho,W) < 0$. The equation $F(\pi;\ \rho,W) = 0$ therefore has a unique root $\pi_1(\rho,W)$ in $[1/2,p)$. Given this uniqueness, the continuity of $\pi_1(.,.)$ is a straightforward consequence of the continuity of F.

Finally, since $F(\pi_1(\rho,W);\ \rho,W) = 0$, $V_1(\rho,W) = V_{0T}(\pi_1(\rho,W);\rho,W)$ is also equal to $V_{0L}(\pi_1(\rho,W);\rho,W)$. Multiplying one of these expressions by p, the other by 1-p, and subtracting therefore leads to two equivalent ways of rewriting $V_1(\rho,W)$:

$$(7.13)\qquad V_1(\rho,W) = \frac{p\ U(1,\pi_1) - (1-p)\ U(1,1-\pi_1)}{2p - 1} + \beta\ W\left[\frac{p\rho}{\pi_1}\right]$$

$$(7.14)\qquad V_1(\rho,W) = \frac{p\ U(1,1-\pi_1) - (1-p)\ U(1,\pi_1)}{2p - 1} + \beta\ W\left[\frac{(1-p)\rho}{1-\pi_1}\right]$$

where π_1 stands for $\pi_1(\rho,W)$. Consider now $(\rho^1,W^1) \geq (\rho^2,W^2)$, and let $\pi_1{}^i = \pi_1(\rho^i,W^i)$, for $i = 1,2$. If $\pi_1{}^1 \geq \pi_1{}^2$, by (7.14), Property 2.2 and the fact that W is nondecreasing, $V_1(\rho^1,W^1) \geq V_1(\rho^2,W^2)$. If $\pi_1{}^1 < \pi_1{}^2$, by (7.13) and the same argument, $V_1(\rho^1,W^1) \geq V_1(\rho^2,W^2)$. Q.E.D.

Proof of Lemma 4.4:

We first show that the unique solution to (4.13) is $\pi^*(\rho,W) = \text{Max}\ \{\rho p+(1-\rho)(1-p),\ \pi_1(\rho,W)\}$. Let $I(\rho) \equiv [\rho p + (1-\rho)(1-p),p]$. When $\pi_1(\rho,W) \in I(\rho)$, Lemma 7.1 implies that $F(.;\ \rho,W)$ decreases on $I(\rho)$, from $F(\rho p+(1-\rho)(1-p);\ \rho,W) \geq 0$ to $F(p;\ \rho,W) < 0$, so the only solution to

(4.13) occurs for $F(\pi;\ \rho,W) = 0$, i.e. $\pi^*(\rho,W) = \pi_1(\rho,W)$. When $\pi_1(\rho,W) < \rho p+(1-\rho)(1-p)$, $F(\pi;\ \rho,W) < 0$ on $I(\rho)$, so the only solution is $\rho p+(1-\rho)(1-p)$.

The continuity of $\pi^*(\rho,W)$ then follows from that of $\pi_1(\rho,W)$; in turn, it implies that $V_T^*(\rho,W) \equiv V_{0T}(\pi^*(\rho,W);\ \rho,W)$, $V_L^*(\rho,W) \equiv V_{0L}(\pi^*(\rho,W);\ \rho,W)$, and $T(\rho;W) \equiv \max\ [V_T^*(\rho,W),\ V_L^*(\rho,W)]$ are all continuous in (ρ,W). In the case where $\pi^*(\rho,W) = \pi_1(\rho,W) \geq \rho p + (1-\rho)(1-p)$, we have: $T(\rho,W) = V_1(\rho,W)$ with $V_1(.,.)$ defined in Lemma 7.1. In the case where $\pi^*(\rho,W) = \rho p +(1-\rho)(1-p)$, we have $F(\pi^*;\ \rho,W) \leq 0$ or $V_T^*(\rho,W) \leq V_L^*(\rho,W) = T(\rho,W)$. Thus: $T(\rho;\ W) = V_2(\rho,W)$, where we define $V_2(\rho,W) = V_{0L}(\rho p+(1-\rho)(1-p);\ \rho,W)$, i.e.:

$$\begin{aligned} V_2(\rho,W) &= U(1,1-\rho p-(1-\rho)(1-p)) \\ &\quad + \beta p\ W\left[\frac{(1-p)\rho}{1-\rho p-(1-\rho)(1-p)}\right] + \beta(1-p)\ W\left[\frac{p\rho}{\rho p+(1-\rho)(1-p)}\right] \end{aligned} \tag{7.15}$$

Note that $V_2(\rho,W)$ is nondecreasing in (ρ,W), due to Property 2.2, the fact that $W \in C_+$ and that its arguments in the last two terms are increasing functions of ρ. Summarizing, $T(\rho,W)$ is continuous and satisfies:

$$T(\rho;W) = V_2(\rho,W).1_{\pi^*(\rho,W)\leq\rho p+(1-\rho)(1-p)} + V_1(\rho,W).1_{\pi^*(\rho,W)>\rho p+(1-\rho)(1-p)}$$

where V_1 and V_2 are both nondecreasing in their arguments. To prove that T is nondecreasing in (ρ,W), consider $(\rho^1,W^1) \geq (\rho^2,W^2)$. If $\pi^{*1} = \pi_1{}^1 > \rho^1 p + (1-\rho^1)(1-p)$ and $\pi^{*2} = \pi_1{}^2 > \rho^2 p + (1-\rho^2)(1-p)$, then:

$$T(\rho^1,W^1) = V_1(\rho^1,W^1) \geq V_1(\rho^2,W^2) = T(\rho^2,W^2).$$

If $\pi^{*1} = \rho^1 p + (1-\rho^1)(1-p)$ and $\pi^{*2} = \rho^2 p + (1-\rho^2)(1-p)$, the same argument holds, using V_2 instead of V_1. If $\pi^{*1} = \pi_1{}^1 \geq \rho^1 p +$

$(1-\rho^1)(1-p)$ and $\pi^{*2} = \rho^2 p + (1-\rho^2)(1-p) \le \pi_1{}^2$, by continuity there exists (ρ^3, W^3), with $(\rho^1, W^1) \ge (\rho^3, W^3) \ge (\rho^2, W^2)$ and $\pi^{*3} = \pi_1{}^3 = \rho^3 p + (1-\rho^3)(1-p)$. Now:

$$T(\rho^1, W^1) = V_1(\rho^1, W^1) \ge V_1(\rho^3, W^3) = V_2(\rho^3, W^3) \ge V_2(\rho^2, W^2) = T(\rho^2, W^2).$$

The case $\pi^{*1} = \rho^1 p + (1-\rho^1)(1-p)$ and $\pi^{*2} = \pi_1{}^2$ is treated similarly.

Q.E.D.

Proof of Theorem 4.5:

By Lemma 4.4, $T(.)$ maps C_+ continuously into itself, and is nondecreasing in W. Furthermore, by construction: $T(W + c) = T(W) + \beta c$, for any constant c. By the standard argument of Blackwell [1965], T is a contraction mapping, and since C_+ with the sup norm is complete, it has a unique fixed point. Q.E.D.

Proof of Proposition 4.6:

The continuity of $\pi(\rho) = \pi^*(\rho, W(\rho))$, where $W \in C_+$ is the equilibrium value function, results from Lemma 4.4. By (4.12): $\pi(1) = p$, so (7.12) yields: $F(\pi(1); 1, W) = U(1,p) - U(1, 1-p) < 0$. By continuity of F, π and W, this also holds on some interval $(\bar{\rho}, 1]$. Thus by definition, $\pi(\rho) = \rho p + (1-\rho)(1-p)$, and $q(\rho) = 0$ on $(\bar{\rho}, 1]$. On $[0, \bar{\rho})$ (in fact on $[0,1)$), (4.8) implies:

$$(2p-1)q(\rho) = (\pi(\rho) - p\rho)/(1-\rho) - (1-p)$$

so that q is continuous on all of $[0,1]$. Finally, $F(1/2; 0, W) = 0$, so $\pi(0) = \pi^*(0, W) = 1/2$, and the above formula yields $q(0) = 1/2$. Q.E.D.

Proof of Proposition 4.7:

Recall that $\pi(\rho) = \max\{\rho p + (1-\rho)(1-p), \pi_1(\rho, W)\}$ for all ρ. Let $\rho > \rho'$ and denote $\pi(\rho) = \pi$, $\pi(\rho') = \pi'$, $\pi_1(\rho, W) = \pi_1$, $\pi_1(\rho', W) =$

π_1'. Suppose first that $(\pi,\pi') = (\pi_1,\pi_1')$. If $\pi_1 \geq \pi_1'$, (7.13) together with Property 2.2 and the monotonicity of W, i.e. $V_1(\rho,W) \geq V_1(\rho',W)$, imply that $p\rho/\pi_1 > p\rho'/\pi_1'$; moreover $(1-p)\rho/(1-\pi_1) > (1-p)\rho'/(1-\pi_1')$ trivially. If $\pi_1 < \pi_1'$, a similar reasoning applies, using (7.14).

Assume now $\pi = \pi_1$ and $\pi' = p\rho'+(1-p)(1-\rho') < \pi_1'$. Then $\pi' \leq \pi$, so $(1-p)\rho'/(1-\pi') \leq (1-p)\rho/(1-\pi)$; moreover:

$$F(\pi';\ \rho',W) - F(\pi;\ \rho,W) = F(\pi';\ \rho',W) < 0.$$

From the definition (7.12) of F, the fact that $\pi' \leq \pi_1$, Property 2.2 and the monotonicity of W, we get $p\rho'/\pi' < p\rho/\pi$. Conversely, let now $\pi = p\rho + (1-p)(1-\rho) < \pi_1' = \pi'$; then $p\rho/\pi > p\rho'/\pi'$ and the other inequality is again obtained from (7.12). Finally, both $\rho/[\rho p + (1-\rho)(1-p)]$ and $\rho/[1-\rho p - (1-\rho)(1-p)]$ are increasing in ρ, so the result holds when $\pi < \pi_1$ and $\pi' < \pi_1'$. Q.E.D.

<u>Proof of Theorem 5.1</u>:

Define for ρ in $[0,1]$: $f(\rho) = E(\rho_{t+1}|\rho_t = \rho,H)$. By (5.1):

(7.16) $\quad f(\rho) \geq \rho$, with strict inequality for $\rho \in (0,1)$.

When the journalist is honest, $(1-\rho_t)_{t\in N}$ is a positive supermartingale, and therefore (Neveu [1975], p. 27) converges almost surely to a positive random variable $1-\rho_\infty$. Thus ρ_t (with distribution $d\mu_t$) converges almost surely to ρ_∞ (with distribution $d\mu_\infty$). For all t:

$$E(\rho_{t+1}|\rho_t=\rho,H) = \int_0^1 E(\rho_{t+1}|\rho_t=\rho,H)\ d\mu_t(\rho) = \int_0^1 f(\rho)\ d\mu_t(\rho)$$

Taking limits as t tends to $+\infty$: $E(\rho_\infty) = \int_0^1 f(\rho)\ d\mu_\infty(\rho)$

Condition (7.16) then requires that $\rho_\infty = 0$ or $\rho_\infty = 1$ almost surely,

otherwise one would have $E(\rho_\infty) > \int_0^1 \rho \, d\mu_\infty(\rho) = E(\rho_\infty)$. Thus $d\mu_\infty$ has all its mass concentrated on $\{0,1\}$; we now show that there can in fact be no positive mass on 0. Consider the process $\{y_t = 1/\rho_t\}_{t\epsilon N}$. By (4.9) it is clearly a martingale, and therefore also a positive supermartingale. By the maximal inequality (Neveu [1975], p. 23), $\sup_{t\epsilon N}(y_t)$ is almost surely finite, therefore $\inf_{t\epsilon N}(\rho_t) = (\sup_{t\epsilon N}(y_t))^{-1}$ is almost surely strictly positive. Hence the result for the case of the honest journalist. A similar argument holds for the opportunistic journalist, because $\{\rho_t\}_{t\epsilon N}$ is then a (strict) supermartingale and $\{z_t = 1/(1-\rho_t)\}_{t\epsilon N}$ a martingale. Q.E.D.

Proof of Proposition 6.1:

Using symmetry, let s = 1. By (7.7) (ommitting the superscript "+"):

$$(7.17) \qquad \frac{\partial^2 U(1,\pi)}{\partial\alpha \, \partial\pi} = \int_{1-p}^{p} \left[\hat{U}' . \frac{\partial^2 P^*}{\partial\alpha \, \partial\pi} + \hat{U}'' . \frac{\partial P^*}{\partial\alpha} . \frac{\partial P^*}{\partial\pi}\right] dL(\bar{\pi}),$$

A sufficient condition for α to satisfy (6.1) is therefore:

$$(7.18) \quad \hat{U}'(P^*) . \frac{\partial^2 P^*}{\partial\alpha \, \partial\pi} + \hat{U}''(P^*) . \frac{\partial P^*}{\partial\alpha} . \frac{\partial P^*}{\partial\pi} < 0, \text{ for all } \pi, \bar{\pi}.$$

With logarithmic utility, the solution to (7.2) is: $B_a(w_a, P, \pi_a) = w_a . (\pi_a - P)/(P(1-P))$, and (7.5) leads to: $P^*(\pi,\bar{\pi};\alpha) = \alpha\,\pi + (1-\alpha)\bar{\pi}$. Moreover, $B(P) = (p-P)/(P(1-P))$ and: $\hat{U}(P) = u_0(w_0) + p\text{Log}(p/P) + (1-p)\text{Log}((1-p)/(1-P))$, so (7.18) becomes:

$$(7.19) \qquad \alpha\ (\pi - \bar{\pi}) \left[\frac{p}{P^{*2}} + \frac{1-p}{(1-P^*)^2}\right] < \frac{p - P^*}{P^*(1-P^*)}$$

which always holds for $\pi \le \bar{\pi}$. Let now $\pi > \bar{\pi}$, and rewrite (7.19) as:

$$\alpha\ (\pi-\bar{\pi})[P^{*2} + p(1-2P^*)] < p\ P^*(1-P^*) - P^{*2}(1-P^*)$$

or: $$p[P^* - \alpha(\pi-\bar{\pi}) - P^*(P^* - 2\alpha(\pi-\bar{\pi}))] > P^{*2}\ (1-\bar{\pi})$$

i.e.: $\quad p[\bar{\pi}(1-\bar{\pi}) + \alpha^2(\pi-\bar{\pi})^2\ ] > (1-\bar{\pi})[\bar{\pi}^2 + 2\alpha\bar{\pi}(\pi-\bar{\pi}) + \alpha^2(\pi-\bar{\pi})^2]$

or: $\quad \alpha^2[(\pi-\bar{\pi})^2(p-1+\bar{\pi})] - 2\alpha\bar{\pi}(1-\bar{\pi})(\pi-\bar{\pi}) + (p-\bar{\pi})\bar{\pi}(1-\bar{\pi}) > 0.$

This second-degree polynomial in α has two positive roots, since $\pi-\bar{\pi} > 0$. The smallest one is:

$$\alpha_- = \frac{\bar{\pi}(1-\bar{\pi}) - \sqrt{\bar{\pi}(1-\bar{\pi})p(1-p)}}{(\pi - \bar{\pi})\ (p - (1-\bar{\pi}))} \tag{7.20}$$

which is clearly a decreasing function of π. Therefore:

$$\alpha_- > \frac{\bar{\pi}(1-\bar{\pi}) - \sqrt{\bar{\pi}(1-\bar{\pi})p(1-p)}}{(p - \bar{\pi})\ (p - (1-\bar{\pi}))} = \frac{z - \sqrt{yz}}{z - y} \tag{7.21}$$

where $z = \bar{\pi}(1-\bar{\pi}) > p(1-p) = y$. But $(z - \sqrt{yz})/(z - y) > 1/2$ for all $z > y$, so that $\alpha < 1/2$ implies (7.21), hence (7.19) and (6.1).

Finally, (6.1) implies that $U(1,\pi;\alpha) - U(1,1-\pi;\alpha)$ decreases in α, for all $\pi > 1/2$. By (7.12), this implies that $\partial\pi_1(\rho,\bar{W};\alpha)/\partial\alpha < 0$ for all $(\rho,\bar{W})$ in $[0,1] \times C_+$; the same thus holds with weak inequality for $\pi^*(\rho,\bar{W};\alpha) = \max\ \{\rho p + (1-\rho)(1-p),\ \pi_1(\rho,\bar{W};\alpha)\}$. In equilibrium, $\pi(\rho;\alpha) = \pi^*(\rho,W(\rho);\alpha)$, and the equilibrium value function W is independent of the current state α of α_t (the α_t's being i.i.d.); therefore $\pi(\rho;\alpha)$ is non-increasing in α. Finally, $(2p-1)q(\rho;\alpha) = \max\ [(\pi(\rho;\alpha)-p\rho)/(1-\rho) - (1-p),\ 0]$, so $\partial q(\rho;\alpha)/\partial\alpha \le 0$, with strict inequality unless $q(\rho;\alpha) = 0$. Q.E.D.

<u>Proofs of the formulas of Section 6b (combined speculative schemes)</u>:

Define, for all $(B_1,B_2;P)$ in $R^2 \times [1-p,p]$, the function:

$$\Phi(B_1,B_2;P) = p\ u_0(w_0+(P-1/2)B_1+(1-P)B_2) + (1-p)\ u_0(w_0+(P-1/2)B_1-PB_2)$$

Its partial derivatives will be denoted as Φ_i or Φ_{ij}, $i,j = 1,2,3$. It is easily checked that Φ is strictly concave in (B_1,B_2). Then:

(7.22) $$U^*(1,\pi) = \max_{B_1} \int_{1-p}^{p} \Phi(B_1, B_2^*(w_0+(P-1/2)B_1,P);P)\ dM_\pi(P) \equiv \Gamma(B_1)$$

where $B_2^* = B_2^*(w_0+(P-1/2)B_1,P)$ is, by (7.2), the solution to:

(7.23) $$\Phi_2(B_1,B_2^*;P) = 0, \text{ for all } B_1 \text{ and } P.$$

We first show that Γ is concave in B_1. Since $\Phi_2 = \Phi_{12} \equiv 0$ by (7.23):

(7.24) $$\Gamma''(B_1) = \int_{1-p}^{p} \Phi_{11}(B_1,B_2^*;P)\ dM_\pi(P) < 0$$

by concavity of Φ. The first-period problem (7.22) therefore has a unique solution $B_1^* = B_1^*(\pi)$; provided that 1/2 is in the interior of the distribution M_π's support, B_1^* is finite, and is the solution to:

(7.25) $$\int_{1-p}^{p} \Phi_1(B_1^*,B_2^{**};P)\ dM_\pi(P) = 0,$$

where the function $B_2^*(w_0+(P-1/2)B_1^*,P)$ is abbreviated as B_2^{**}.

Moreover:

(7.26) $$U^*(1,\pi) = \int_{1-p}^{p} \Phi(B_1^*,B_2^{**};P)\ dM_\pi(P)$$

Integrating by parts and using (7.23) yields:

(7.27) $$U^*(1,\pi) = \Phi(B_1^*,0;p) - \int_{1-p}^{p} \Phi_3(B_1^*,B_2^{**};P)\ M_\pi(P)\ dP$$

By Property 2.1, $M_\pi(P)$ is (for all P) monotonic in π, hence almost everywhere differentiable (a.e.d.) in π, with a derivative $dM_\pi(P)/d\pi \leq 0$. By (7.27), $U^*(1,\pi)$ is also a.e.d. in π, and, using $\Phi_{23} = 0$:

(7.28) $$\partial U^*(1,\pi)/\partial\pi = - \int_{1-p}^{p} \Phi_3(B_1^*,B_2^{**};P)\ (dM_\pi(P)/d\pi)\ dP$$

$$+ B_1^*(\pi)\ [\Phi_1(B_1^*,0;p) - \int_{1-p}^{p} \Phi_{13}(B_1^*,B_2^{**};P)\ M_\pi(P)\ dP] ,$$

But integrating (7.25) by parts, using $\Phi_{12} = 0$, yields:

$$(7.29)\qquad \Phi_1(B_1^*,0;p) = \int_{1-p}^{p} \Phi_{13}(B_1^*,B_2^{**};P)\ M_\pi(P)\ dP$$

so that the last two terms in (7.28) cancel out, leaving only the right-hand-side of (6.3). There may also be points π where $M_\pi(P)$ is not differentiable -hence neither is $U^*(1,\pi)$; at such points, the same expressions and computations hold, but now $\partial U^*(1,\pi)/\partial\pi$ and $dM_\pi(P)/d\pi$ represent generalized functions. Thus in all cases, if (6.3) holds (in the appropriate sense) $\partial U^*(1,\pi)/\partial\pi < 0$ (in the appropriate sense) implying (in all cases) that $U^*(1,\pi)$ decreases with π. Q.E.D.

Case of logarithmic utilities:

As seen in the proof of Proposition 6.1, $\bar{P}(\pi) = P^*(\pi,\bar{\pi};\alpha) = \alpha\pi + (1-\alpha)\bar{\pi}$ and $B_2^* = B_0(w_0+(P-1/2)B_1,P) = (w_0+(P-1/2)B_1)$. $(p-P)/[P(1-P)]$ for all B_1 and P; replacing in (7.26) with $B_1 = B_1^*$ yields (6.5). Q.E.D

Proof of Proposition 6.2:

With logarithmic utility and the distributional assumption made on $\bar{\pi}$, (7.25) takes the form:

$$(7.30)\qquad \int_{\underline{P}(\alpha,\pi)}^{\bar{P}(\alpha,\pi)} \frac{P - 1/2}{w_0 + (P-1/2)B_1^*}\, dP = 0.$$

When $\alpha = 0$, $\underline{P}(\alpha,\pi) = 1-p$ and $\bar{P}(\alpha,\pi) = p$ (for all π) are symmetric around 1/2, so the only solution is $B_1^* = 0$, and (6.3) is satisfied. By continuity, it still holds for small values of α, hence Proposition 6.2; a Taylor expansion of (7.30) yields (6.6). Q.E.D.

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Figure 1: Timing of moves and informations.

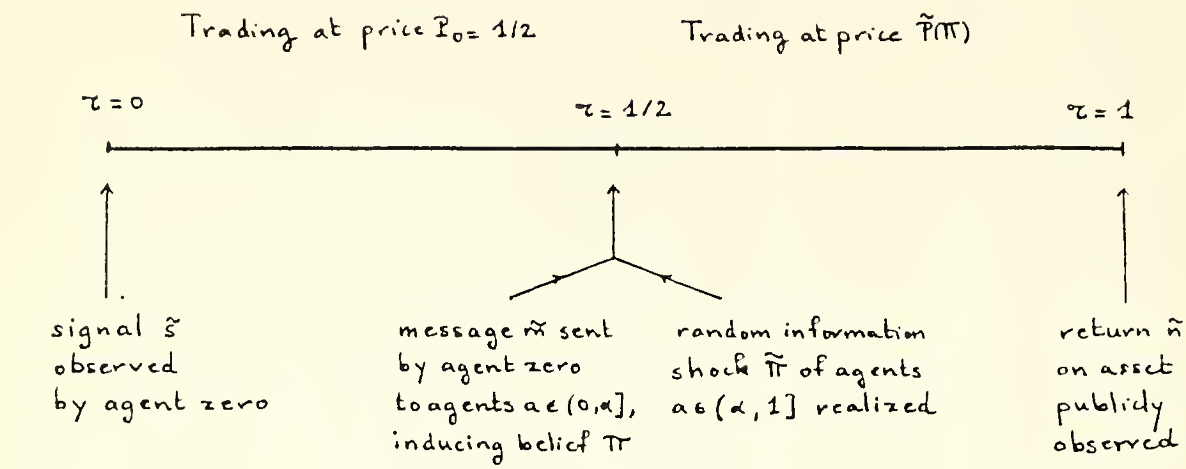

Figure 2: The two market manipulation schemes (for $\tilde{s} = +1$); R_i denotes the expected rate of return, as of $\tau = 0$, of agent zero, for speculation in subperiod $i = 1,2$ when he decides to report truthfully (R_{iT}) or to lie (R_{iL}).